THE NUMBER π AS A STRUCTURAL INVARIANT OF SELF-CONSISTENT OBSERVATION IN THE OBSERVER-DEPENDENT THEORY OF EVERYTHING (ODTOE)

(Число π как структурный инвариант самосогласованного наблюдения в наблюдатель-зависимой теории всего)

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ABSTRACT

Within the framework of the Observer-Dependent Theory of Everything (ODTOE), which posits the conscious observer as the primary agent of reality formation, this paper investigates the role of the number π as a structural invariant that necessarily arises in self-consistent observation configurations. Five independent mathematical arguments are shown to produce π within the ODTOE formalism: topological (the homotopy type of the closed self-observation loop), spectral (the imaginary part of eigenvalues of the linearized operator near a fixed point), measure-theoretic (the normalization factor of the Gaussian measure on an infinite-dimensional space), dynamical (the oscillation period of the coupled "reality–beliefs" system), and algebraic (Euler's identity as a bridge between discrete and continuous structures). The connection between the transcendence of π and the structural incompleteness of the metatheory (spiral rather than circular dynamics) is investigated; consequences for the interpretation of the Planck constant $\hbar = h/(2\pi)$ are discussed. The ternary architecture of the minimal self-consistent observation act (observer, observed, observation operator) and its relation to Archimedes' lower bound for π are analyzed. It is shown that the strange loop (in Hofstadter's sense) represents the topologically unique mechanism of self-generation of reality without an external agent. Additionally, the role of the golden ratio $\varphi = (1 + \sqrt{5})/2$ as a complementary structural invariant arising from the discrete iterative dynamics of self-reference through the same Banach fixed-point mechanism is investigated. The results formalize the proposition that the presence of π and φ in fundamental physical constants is determined not by the geometry of space, but by the cyclic and iterative nature of the act of observation.

Keywords: number π, golden ratio, theory of everything, observer, self-reference, fixed point, Gaussian measure, strange loop, ODTOE, Euler's formula, coherence, spiral dynamics, Planck constant, Fibonacci numbers.

АННОТАЦИЯ

В рамках наблюдатель-зависимой теории всего (ODTOE), полагающей сознательного наблюдателя основным агентом формирования реальности, исследуется роль числа π как структурного инварианта, закономерно возникающего в самосогласованных конфигурациях наблюдения. Пять независимых математических аргументов — от гомотопического типа замкнутой петли самонаблюдения до тождества Эйлера как моста между дискретными и непрерывными структурами — обнаруживают необходимое присутствие числа π в формализме ODTOE, причём каждый аргумент задействует различный раздел математики (алгебраическая топология, спектральная теория, теория меры, теория динамических систем, абстрактная алгебра). Исследована связь трансцендентности π со структурной неполнотой метатеории (спиральная, а не круговая динамика), обсуждены следствия для интерпретации постоянной Планка $\hbar = h/(2\pi)$. Дан анализ тройственной архитектуры минимального самосогласованного акта наблюдения (наблюдатель, наблюдаемое, оператор наблюдения) и её связи с нижней оценкой Архимеда для π. Показано, что странная петля (в смысле Хофштадтера) представляет топологически единственный механизм самопорождения реальности без привлечения внешнего агента. Дополнительно исследована роль золотого сечения $\varphi = (1 + \sqrt{5})/2$ как комплементарного структурного инварианта, возникающего из дискретной итеративной динамики самореференции через тот же механизм теоремы Банаха, который обосновывает существование неподвижной точки самонаблюдения. Результаты формализуют положение о том, что присутствие π и φ в фундаментальных физических постоянных обусловлено не геометрией пространства, а циклической и итеративной природой акта наблюдения.

Ключевые слова: число π, золотое сечение, теория всего, наблюдатель, самореферентность, неподвижная точка, гауссова мера, странная петля, ODTOE, формула Эйлера, когерентность, спиральная динамика, постоянная Планка, числа Фибоначчи.

I. INTRODUCTION

1.1. Context and Problem Statement

The number π, defined as the ratio of a circle's circumference to its diameter, permeates theoretical physics far more deeply than its geometric origin would suggest. The presence of π in theoretical physics is not limited to trigonometry: it enters the definition of the reduced Planck constant ($\hbar = h/(2\pi)$), governs the phase of the wave function through the factor $2\pi i$ in the Schrödinger equation, determines the minimum uncertainty product ($\Delta x\,\Delta p \geq h/(4\pi)$), and normalizes probability distributions through the Gaussian factor $\sqrt{2\pi}$ [1, 2]. The standard explanation attributes the presence of π to the periodicity of trigonometric and exponential functions: wave phenomena are described by sines and cosines whose full period is 2π. Such an interpretation, however, leaves unanswered a question of a deeper order: why do cyclic structures occupy a privileged position in physical description?

The Observer-Dependent Theory of Everything (ODTOE) [3] offers an alternative perspective. The central axiom of ODTOE states: the observer and the observed mutually constitute each other in the act of observation, and the result of observation is determined by the composite system "observer + object" (formula A.1 of the main article [3]). Proposition 4 of the main article [3] establishes the existence of a self-consistent configuration $\Psi^* = \Phi(\Psi^*)$, in which the field of potential states generates the observer who constitutes that same configuration — a fixed point of the self-observation mapping. Proposition 3 [3] establishes the self-referential architecture of the theory: ODTOE belongs to the set $\mathbb{T}$ of theories of everything, whose structure it itself determines.

The present work poses the following task: to show that the number π necessarily appears in any formalism satisfying the axiom of constructive observation (A) and the self-consistency condition (Proposition 4), thereby establishing its status as a structural invariant of observation rather than an externally imported geometric constant.

1.2. Structure of the Paper

Section II briefly reproduces the necessary elements of the ODTOE formalism. Section III contains five independent arguments for the appearance of π in the structure of self-consistent observation. Section IV is devoted to consequences — the connection between the transcendence of π and spiral dynamics, the ternary architecture of the minimal observation act, and the uniqueness of the strange loop as a self-generation mechanism. Section V discusses the interpretation of the Planck constant. Section V-bis demonstrates that the golden ratio φ serves as a complementary structural invariant generated by the discrete iterative dynamics of self-reference. Section VI contains a discussion of limitations and connections with existing works. Section VII summarizes the results.

II. NECESSARY ELEMENTS OF THE ODTOE FORMALISM

To ensure self-containedness of the exposition, we reproduce the key definitions and formulas of ODTOE [3] in the notation of the present paper. Formula numbers with primes (A.1′, D1.1′, 4.4′, U4.1′, 4.5′) correspond to formulas of the main article, written using the unified notation of this work.

Axiom (A). The observer and the observed mutually constitute each other in the act of observation. Reality is defined by the formula:

$$R = \hat{O}(\Psi) \tag{A.1′}$$

where $\hat{O}$ is the observation operator, dependent on the properties of the observer, $\Psi \in \mathcal{H}$ is the field of potential states (an element of the infinite-dimensional Hilbert space $\mathcal{H}$, formalized rigorously as a rigged Hilbert space [4]).

Observation operator. Each observer O_i is described by a state vector:

$$O_i = (B_i, A_i, H_i) \tag{π-2.1}$$

where $B_i \in [0, 1]$ is the contextual belief (cognitive coherence), A_i is the archetype of the attention focus, H_i is the observation history.

Contextual belief.

$$B(O, C) = F(O, C)^{w_1} \cdot E(O, C)^{w_2} \cdot (1 - \sigma(O, C))^{w_3} \cdot \Lambda(O, C)^{w_4} \tag{D1.1$'$}$$

where F is the attention focus, E is the emotional coherence, σ is the internal contradiction, Λ is the empirical reinforcement (formula D1.1 [3]).

Reconfiguration dynamics.

$$\frac{dC}{dt} = -\frac{\alpha}{I(C) + \varepsilon} \cdot \nabla U(C) + \eta(t) \tag{4.4$'$}$$

with a stochastic term whose variance $D(\eta) = D_0 \cdot (1 - S)$ decreases as coherence S grows (formula 4.4 [3]).

Self-observation mapping.

$$\Phi(\Psi) = \iota(\hat{O}_\Psi(\Psi)) \tag{U4.1$'$}$$

where $\iota : \mathbb{C} \to \mathcal{H}$ is the embedding operator. The fixed point $\Psi^* = \Phi(\Psi^*)$ defines the self-consistent configuration (Proposition 4 [3]).

Coherence.

$$S = 1 - \frac{2}{n(n-1)} \sum_{i<j} |B_i - B_j| \tag{4.5$'$}$$

III. FIVE ARGUMENTS FOR THE APPEARANCE OF THE NUMBER π

3.1. Topological Argument: Homotopy Type of the Self-Observation Loop

Consider the self-observation mapping $\Phi : \mathcal{H} \to \mathcal{H}$, defined in (U4.1′). The sequence $\Psi \to \hat{O}(\Psi) \to R \to \iota(R) \to \Psi'$ defines a closed trajectory in $\mathcal{H}$ under the condition $\Psi' = \Psi$, i.e., at the fixed point. Denote this trajectory as $\gamma : [0, 1] \to \mathcal{H}$, $\gamma(0) = \gamma(1) = \Psi^*$. Then γ defines an element of the fundamental group $\pi_1(\mathcal{H}, \Psi^*)$.

In the finite-dimensional case, a closed path in Euclidean space is contractible ($\pi_1(\mathbb{R}^n)$ is trivial). However, the self-observation loop contains an essential feature:

the operator $\hat{O}$ performs a projection (dimension reduction), while ι performs an embedding (dimension expansion). The irreversibility of the operator $\hat{O}$ (projection destroys information about the orthogonal component) leads to the effective dynamics being restricted to a subspace with nontrivial topology; a closed path in such a subspace turns out to be non-contractible, which is consistent with the irreversibility of the act of observation (wave function collapse in standard quantum mechanics).

Let us formalize. Suppose $\mathcal{H}$ decomposes into a direct sum $\mathcal{H} = \mathcal{H}_{\text{obs}} \oplus \mathcal{H}_{\text{ort}}$, where $\mathcal{H}_{\text{obs}}$ is the subspace actualized by observation, $\mathcal{H}_{\text{ort}}$ is the orthogonal complement. The operator $\hat{O}$ projects Ψ onto $\mathcal{H}_{\text{obs}}$, the operator ι embeds the result back into $\mathcal{H}$. The effective dynamics is restricted to $\mathcal{H}_{\text{obs}}$, and the trajectory γ describes a closed path in a subspace homotopically equivalent to the circle S^1. The fundamental group of the circle is $\pi_1(S^1) = \mathbb{Z}$ [5], and the generator is one full traversal of length 2π (at unit radius). Thus, the topological constant of closure of the self-observation loop is 2π.

It should be noted that the homotopic equivalence of the effective subspace to the circle S^1 is adopted here as an assumption (D-Top), motivated by the one-dimensionality of the belief parameter B, which governs the operator $\hat{O}$. When the parameter space is expanded (including A, H), the effective topology may become more complex; analysis of this case constitutes an open problem.

Strictly speaking, it is necessary to show that the projection-embedding dynamics ($\hat{O} \circ \iota$) induces an effective restriction to a submanifold of fixed dimension with a nontrivial fundamental group. In the current formalization, this statement relies on assumption (D-Top), according to which the parameter $B \in [0, 1]$ specifies the single governing degree of freedom of the operator $\hat{O}$. Since the observer operator is defined by the triple (B, A, H), a complete justification of D-Top would require demonstrating that, with A and H fixed, the effective dynamics projects onto a one-dimensional cyclic subspace. An indirect argument in favor of effective one-dimensionality comes from the analogy with the golden ratio dynamics: the mapping $f(x) = 1 + 1/x$ on the real line generates one-dimensional dynamics with a nontrivial attractor structure (Section V-bis), which points to the general character of reduction of self-referential systems to one-dimensional iterative processes. This question is identified as an open problem.

3.2. Spectral Argument: Eigenvalues of the Linearized Operator

Let Φ be Fréchet differentiable [6] in a neighborhood of the fixed point Ψ^*. Denote $D\Phi|_{\Psi^*} = L$ — the linearization of the operator Φ. The stability of Ψ^* is determined by the spectrum of L.

For a contractive mapping (Banach's theorem [7]), the spectral radius $r(L) < 1$. The eigenvalues of L are complex in the general case:

$$\lambda_j = |\lambda_j| \cdot e^{i\theta_j} \qquad (\pi\text{-}3.1)$$

where $|\lambda_j| < 1$ ensures damping, and θ_j determines the angular frequency. The iterative dynamics in the neighborhood of Ψ^* takes the form:

$$\delta\Psi_{n+1} = L \cdot \delta\Psi_n \approx \sum_j c_j\, |\lambda_j|^n\, e^{in\theta_j}\, v_j \qquad (\pi\text{-}3.2)$$

where v_j are the eigenvectors, c_j are the expansion coefficients of the initial deviation.

The condition for the system to return to its original phase: $n\theta_j = 2\pi m$ for integers n, m. The full phase cycle is determined by the relation $\theta = 2\pi m/n$, where the number 2π sets the length of the full traversal in phase space. The number π is present in the closure condition for any nontrivial ($\theta \neq 0$) oscillatory regime: whatever the eigenfrequency θ_j, the return period $T_j = 2\pi/\theta_j$ contains the factor 2π. It should be noted that the factor 2π in the expression for the period is due to the standard convention in which a full revolution in phase space is measured in radians. The choice of radian measure, however, is not an arbitrary convention: it is consistent with the measure-theoretic argument (Section 3.3), where $\sqrt{2\pi}$ arises from the fundamental Gaussian integral without invoking angular conventions, which confirms the structural rather than conventional character of the appearance of π. It is noteworthy that in spectral analysis of discrete interaction matrices (in particular, the Fibonacci matrix $\mathbf{M} = \left(\begin{smallmatrix}1&1\\1&0\end{smallmatrix}\right)$), the largest eigenvalue equals the golden ratio $\varphi = (1+\sqrt{5})/2$, rather than containing π; this points to the complementarity of two structural invariants: π governs the continuous phase dynamics, while φ governs discrete recurrent structures (Section V-bis).

Here the assumption (D-Fr) of Fréchet differentiability of Φ is used. In the main ODTOE article [3, Section II], the specification of the analytical properties of the operator $\hat{O}$ is designated as an open problem. For any specific choice of $\hat{O}$ with a complex spectrum, the imaginary part necessarily includes the factor 2π.

3.3. Measure-Theoretic Argument: Normalization of the Gaussian Measure

The space of potential states $\mathcal{H}$ is infinite-dimensional (Axiom A). To define probabilities in ODTOE (postulate P4: $P(E \mid B) = B^k$), which extends the scope of the Born rule of standard quantum mechanics [8], a measure on $\mathcal{H}$ is needed that allows normalization of distributions.

By Minlos' theorem [9], a σ-additive Gaussian measure μ_G exists on a nuclear space. In a finite-dimensional projection onto $\mathbb{R}^n$, its density is:

$$d\mu_G = (2\pi)^{-n/2} \cdot \exp\left(-\frac{\|x\|^2}{2}\right) dx_1 \ldots dx_n \qquad (\pi\text{-}3.3)$$

The normalization factor $(2\pi)^{-n/2}$ ensures $\int d\mu_G = 1$. It is generated by the fundamental Gaussian integral:

$$\int_{-\infty}^{\infty} \exp\left(-\frac{x^2}{2}\right) dx = \sqrt{2\pi} \qquad (\pi\text{-}3.4)$$

proved by Laplace through transition to polar coordinates [10]. The number π arises here from the requirement of finite norm: the standard proof by Laplace [10] uses transition to polar coordinates, but in the context of ODTOE the essential point is that the necessity of the π-containing normalization factor is dictated not by the spatial geometry of the observed world, but by the structure of the infinite-dimensional space of potential states $\mathcal{H}$. For this space to admit a probabilistic interpretation, the normalization factor must contain $\sqrt{2\pi}$ per degree of freedom.

In ODTOE, this has a concrete meaning: the probability $P(c_j \mid O_i)$ that observer O_i actualizes configuration c_j from an infinite set of alternatives (formula 4.3 [3]) is finite only in the presence of a π-containing normalization.

3.4. Dynamical Argument: Oscillations of the Coupled System $R \leftrightarrow B$

In ODTOE, reality R and contextual belief B are connected by feedback (Section 4.5 [3]): $R = F[\{O_i(t)\}, S(t), I(C(t))]$, and $dB/dt = G(B, R(B))$. This nonlinear system generates oscillatory dynamics.

Linearization in the neighborhood of the stationary state (B^*, R^*) yields the system:

$$\frac{d(\delta B)}{dt} = \frac{\partial G}{\partial B}\,\delta B + \frac{\partial G}{\partial R}\,\delta R \qquad (\pi\text{-3.5a})$$

$$\frac{d(\delta R)}{dt} = \frac{\partial F}{\partial B}\,\delta B + \frac{\partial F}{\partial R}\,\delta R \qquad (\pi\text{-3.5b})$$

The characteristic equation $\det(\mathbf{A} - \lambda\mathbf{I}) = 0$ for the Jacobian matrix $\mathbf{A}$ admits complex roots $\lambda = \alpha \pm i\omega$ when the discriminant is negative. The oscillation frequency ω determines the period:

$$T = \frac{2\pi}{\omega} \qquad (\pi\text{-3.6})$$

Friedmann and Hagen [11] demonstrated that comparing the variational estimate of energy levels of the hydrogen atom with the exact quantum-mechanical solution in the limit of large orbital angular momentum quantum numbers reproduces the Wallis formula:

$$\frac{\pi}{2} = \prod_{n=1}^{\infty} \frac{4n^2}{4n^2 - 1} \qquad (\pi\text{-3.7})$$

This result points to a connection between the number π and quantum oscillatory dynamics at a fundamental level — a connection that in the context of ODTOE is interpreted as a manifestation of the oscillations of the system $R \leftrightarrow B$.

3.5. Algebraic Argument: Euler's Identity

ODTOE simultaneously operates with discrete structures (a finite number of observers N, discrete observation acts, integer powers in formula P4.1: $P(E \mid B) = B^k$) and continuous ones (a smooth configuration space $\mathbb{C}$, continuous evolution of coherence $S(t)$, differentiable dynamics D1.3).

Euler's identity:

$$e^{i\pi} + 1 = 0 \qquad (\pi\text{-}3.8)$$

unifies five fundamental mathematical constants: e (continuous exponential dynamics, present in equation D1.3 through tanh and the logistic function), i (the imaginary unit, generating the complex spectrum of the operator Φ, Section 3.2), π (the closure invariant, Sections 3.1–3.4), 0 and 1 (the boundary values of the parameter $B \in [0, 1]$).

Euler's identity serves not as an external illustration but as an algebraic identity connecting all key elements of the ODTOE formalism. This argument is conceptual rather than probative in character: it points to structural consistency but is not an independent derivation of π from the axioms of ODTOE.

IV. CONSEQUENCES

4.1. Transcendence of π and Spiral Dynamics of Observation

The number π is transcendental (Lindemann, 1882 [12]): it is not a root of any polynomial with rational coefficients. The decimal expansion of π is neither periodic nor terminating.

In the context of ODTOE, the transcendence of π acquires a substantive interpretation. Proposition 3 of the main article [3] establishes structural incompleteness: the limit $S \to 1$ (a unified configuration) is unattainable, since a complete self-description would require including a description of the description itself. Each act of observation (a loop iteration $\Psi \to \hat{O} \to R \to \iota \to \Psi'$) refines the configuration but does not complete it. The iterative sequence $\Psi_{n+1} = \Phi(\Psi_n)$ converges to Ψ^* (by Banach's fixed-point theorem [7], under the conditions of completeness of the metric space $\mathcal{H}$ and contractivity of the mapping Φ), and the dynamics near Ψ^* is described by a spiral (Section 3.2): $|\lambda| < 1$ ensures approach, $\theta \neq 0$ ensures rotation.

Suppose the angular step θ is rational: $\theta = 2\pi(p/q)$. Then q iterations would return the system exactly to its original phase, and the cycle would close in a finite number of steps. However, this would lead to a finite, fully describable reality — contradicting Proposition 3 (structural incompleteness) and Proposition 2 (unattainability of $S = 1$). Strictly speaking, the closure of a single phase parameter (the angular step θ) does not necessarily entail complete describability of the entire system, since nonlinear effects can generate complex behavior even at rational frequencies. The given argument should be considered in the context of linearized dynamics near the fixed point: it is precisely in this approximation that rationality of θ leads to finite recurrence, which contradicts the principle of inexhaustibility. Full formalization would require analysis of nonlinear regimes, which constitutes the subject of further work. Nevertheless, consistency requires an irrational (and, in particular, transcendental) angular step: a cycle containing π does not close exactly in a finite number of iterations. Each successive approximation to the decimal expansion of π (3; 3.1; 3.14; 3.141; ...) corresponds to a successive refinement of the configuration — iterations of the spiral with increasing but never complete precision.

Thus, the transcendence of π turns out to be not an accidental mathematical property but a formal expression of the inexhaustibility of observation: neither a finite nor an algebraic closure constant is compatible with the self-referential architecture of ODTOE.

4.2. Ternary Architecture of the Minimal Observation Act and Archimedes' Lower Bound

The integer part of π equals 3. This fact has an elementary geometric justification: the sides of a regular hexagon inscribed in a circle of radius r equal r, and its perimeter is $6r$. With diameter $d = 2r$, the circumference $C = \pi d > 6r = 3d$, whence $\pi > 3$ (Archimedes' lower bound [13]).

This bound admits an interpretation within ODTOE. The minimal self-consistent observation act, according to Axiom (A), includes three components: (1) the observer $O = (B, A, H)$, formalized as a state vector; (2) the observed R — a configuration from the space $\mathbb{C}$; (3) the observation operator $\hat{O}$, effecting the mapping $\mathcal{H} \to \mathbb{C}$.

Without any of the three components, self-consistency is impossible: without the observer there is no subject of reduction; without the observed there is no object; without the operator there is no connection between them. Ternarity is the minimal condition for loop closure.

Archimedes' bound $\pi > 3$ is then interpreted as follows: the ratio of the length of the closed observation cycle to its "diameter" (the maximum distance between opposite phases: observer $\leftrightarrow$ observed) exceeds 3, since closure requires a nonlinear (curvilinear) path rather than a rectilinear one. The three-component structure sets the minimal approximation; the exact value $\pi = 3.14159\ldots$ reflects the "curvature" of the act of observation, exceeding the minimal ternarity.

This interpretation is heuristic in character: the formal connection between the number of components of the minimal observation act (three) and the value of Archimedes' lower bound ($\pi > 3$) has not been established deductively and represents a substantive analogy rather than a rigorous consequence of the axiomatics.

4.3. The Strange Loop as a Topologically Distinguished Self-Generation Mechanism

We justify the proposition that the strange loop [14, 15] is the unique mechanism for the emergence of a self-consistent reality without invoking an external agent.

Let a configuration Ψ^* be required that satisfies the conditions: (a) all causes of Ψ^* are internal — Ψ^* is defined through its own components; (b) self-sufficiency — each component is defined through the others; (c) non-contradiction — Ψ^* does not destroy itself.

Condition (a) excludes linear causal chains $A \to B \to C \to \ldots$, which require an external beginning. Condition (b) excludes open structures leading to infinite regress. Condition (c) excludes chaotic configurations without stable structure.

The set of conditions (a)–(c) is mathematically equivalent to the existence of a fixed point of a mapping $\Phi : X \to X$, where X is the space of admissible configurations. The fixed point $\Psi^* = \Phi(\Psi^*)$ satisfies all three conditions: the cause of Ψ^* is the mapping Φ itself acting on Ψ^* (condition a); Ψ^* determines Φ, which determines Ψ^* (condition b); stability is ensured by contractivity or compactness of the image (condition c).

Cahill and Klinger [16] proposed a pregeometric model in which self-referential processes in information systems lead to the spontaneous emergence of three-dimensional spatial structures, confirming the productivity of self-referential constructions for the emergence of spatial order.

Gödel's incompleteness theorem [17] adds a constraint: a self-referential system of sufficient power contains statements that are true but unprovable within the system. In ODTOE, this corresponds to Proposition 3: complete self-description is fundamentally unattainable ($S < 1$). The strange loop not only generates reality but also guarantees its inexhaustibility.

V. π AND THE PLANCK CONSTANT: THE OBSERVER'S SIGNATURE

The Planck constant $\hbar = h/(2\pi)$ contains the factor 2π, traditionally explained as a notational convenience for angular frequencies ($\omega = 2\pi\nu$). This interpretation, however, reduces the appearance of π to a notational convention.

ODTOE allows a nontrivial interpretation to be proposed. Quantization — discretization of a continuous spectrum — is formalized through the selection of discrete eigenvalues from the continuous configuration space. By the argument of Section 3.3, the normalization of distributions on the infinite-dimensional space $\mathcal{H}$ requires a factor containing $\sqrt{2\pi}$. By the argument of Section 3.1, the closure of one full observation cycle has topological length 2π. Division of h by 2π transforms action (a discrete quantity, quantized in units of h) into angular momentum (a quantity associated with one full cycle). The factor 2π in the denominator of $\hbar$ is not a notational convention but the quantitative expression of one full revolution of the self-observation loop: the minimal action h is divided by the length of one cycle 2π, yielding the minimal angular momentum $\hbar$.

The uncertainty relation $\Delta x\,\Delta p \geq \hbar/2 = h/(4\pi)$ contains the factor $4\pi = 2 \times 2\pi$. Formally, it follows from the commutation relation $[\hat{x}, \hat{p}] = i\hbar$ and Robertson's inequality. In the context of the proposed heuristic, the following metaphorical interpretation is admissible: the factor $2\times 2\pi$ corresponds to two conjugate observation acts — one fixes the coordinate, the other fixes the momentum. The impossibility of simultaneously determining both quantities precisely is then explained by the fact that two conjugate observation acts cannot be performed simultaneously — a heuristic consequence of the ternary architecture (Section 4.2), which requires sequential passage through phases.

This interpretation is heuristic in character and is not rigorously derived from the axiomatics of ODTOE. It indicates the direction in which the formal inclusion of π in the structure of the theory may generate substantive predictions.

V-bis. THE GOLDEN RATIO φ AS A COMPLEMENTARY STRUCTURAL INVARIANT

The five arguments of Sections III–IV establish the appearance of π in the ODTOE formalism through the continuous dynamics of the self-observation loop. The present section demonstrates that the discrete iterative dynamics of self-reference generates a second structural invariant — the golden ratio $\varphi = (1 + \sqrt{5})/2$.

V-bis.1. The Fixed Point and Banach's Theorem

Proposition 4 of the main article [3] establishes the existence of a fixed point $\Psi^* = \Phi(\Psi^*)$ of the self-observation mapping, proved through Banach's theorem [7]. The same mechanism generates the golden ratio: the mapping $f(x) = 1+1/x$ is contractive on $[3/2, 2]$ with Lipschitz constant $4/9$, and its unique positive fixed point is φ [25]. The sequence of ratios F_{n+1}/F_n, where F_n are the Fibonacci numbers, constitutes an orbit of f converging to φ. Thus, Banach's theorem [7] — already used in the proof of Proposition 4 — simultaneously generates the golden ratio as an invariant of discrete iterative dynamics.

V-bis.2. Spectral Parallel

The Fibonacci recurrence relation admits a matrix representation through the matrix $\mathbf{M} = \left(\begin{smallmatrix} 1 & 1 \\ 1 & 0 \end{smallmatrix}\right)$ with eigenvalues $\lambda_1 = \varphi$ and $\lambda_2 = -1/\varphi = (1 - \sqrt{5})/2$: the golden ratio is the largest eigenvalue of the fundamental binary interaction matrix. If the argument of Section 3.2 establishes π as an invariant of the continuous spectrum (closure condition $n\theta = 2\pi m$), then φ serves as an invariant of the discrete spectrum. Experimental confirmation: at the quantum critical point of the Ising chain $CoNb_2O_6$, the ratio of the two smallest resonance frequencies of magnetic spins equals $\varphi = 1.618\ldots$, which is a signature of hidden E_8 symmetry [26].

V-bis.3. Measure-Theoretic Limit: Hardy's Probability

Hardy [27] showed that the maximum probability of nonlocal quantum correlation of two particles equals $P_{\text{Hardy}} = \varphi^{-5} \approx 0.09017$. If π normalizes the Gaussian measure in the space of potential states $\mathcal{H}$ (Section 3.3), then φ sets the fundamental probabilistic limit in quantum nonlocality. In the ODTOE formalism, this indicates that self-consistent observation of two entangled subsystems is bounded by a φ-containing limit.

V-bis.4. The KAM Theorem and Maximal Stability

The Kolmogorov–Arnold–Moser theorem [28, 29, 30] establishes that invariant tori with "sufficiently irrational" frequency ratios are stable under small perturbations. The golden ratio, as the number with the worst rational approximations ($\varphi = [1; 1, 1, 1, \ldots]$), ensures maximal stability of orbits. In the context of ODTOE: if the transcendence of π guarantees non-closure of continuous phase trajectories (Section 4.1), then the "maximal irrationality" of φ guarantees maximal stability of non-closed orbits near the fixed point Ψ^*.

V-bis.5. The Self-Referential Equation and the Strange Loop

The equation $\varphi = 1 + 1/\varphi$ is the simplest nontrivial self-referential algebraic equation: the value is defined through itself. This is the exact algebraic analog of Proposition 3 of the main article [3]: $T_{\text{ODTOE}} \in \mathbb{T}$. If Euler's identity (Section 3.5) describes the completeness of the complex structure of ODTOE, then the equation $\varphi = 1 + 1/\varphi$ describes the minimal algebraic self-reference. Binet's formula $F_n = (\varphi^n - \psi^n)/\sqrt{5}$, where $\psi = (1 - \sqrt{5})/2 = -1/\varphi$ [31], explicitly derives the discrete Fibonacci sequence from continuous powers of φ, demonstrating the transition from continuous dynamics to discrete structures — mirroring the Wallis formula [11], in which rational factors generate the transcendental π.

V-bis.6. Complementarity of π and φ

The two structural invariants do not compete but complement each other: π governs the continuous phase dynamics of the self-observation system (rotations, measure normalization, oscillations), while φ governs the discrete iterative dynamics of self-reference (fixed points, recurrent structures, orbital stability). Both invariants are connected through a common mechanism — Banach's fixed-point theorem [7], which ensures the unity of their origin within the axiomatics of ODTOE. The transcendence of π is necessary for the non-closure of continuous phase trajectories (Section 4.1), while the algebraic irrationality of φ is sufficient for the maximal stability of discrete iterative orbits; the requirements of different aspects of the dynamics impose different conditions on the type of irrationality, and both conditions are satisfied.

VI. DISCUSSION

6.1. Connection with Existing Works

The problem of the unexplained prevalence of mathematical structures in physics, posed by Wigner [18], admits a concretization in the context of the present work: the issue is not the “unreasonable effectiveness” of mathematics as such, but the fact that the act of self-consistent observation, formalized mathematically, generates a specific set of structural invariants, of which π is the most elementary.

Penrose’s tripartite ontology [19] — the mathematical world, the physical world, the world of consciousness — resonates with the ternary architecture of the minimal observation act (Section 4.2); however, ODTOE does not postulate the independent existence of a mathematical world: π arises as a property of the self-observation loop, not as an inhabitant of a Platonic universe.

Wheeler [20] proposed the metaphor “it from bit” — the informational foundation of reality. In an earlier work, Wheeler [21] developed the concept of a “participatory universe,” in which the observer does not merely record but participates in the formation of physical reality. The “it from bit” program does not, however, contain a self-closure mechanism: information presupposes a source. ODTOE, through Proposition 4 [3], closes the circuit: information generates the observer, who generates information.

The loop quantum gravity program [22] shares with ODTOE the thesis of the secondary nature of spatial geometry; however, in that approach, the primary entities are discrete quantum structures (spin networks), whereas in ODTOE it is the act of observation as such.

Stepin [23], within the framework of the concept of post-nonclassical rationality, substantiated the necessity of including the cognizing subject in the structure of scientific knowledge, which methodologically precedes the formalization of ODTOE. If classical science excluded the observer, and nonclassical science (quantum mechanics) accounted for the means of observation, then the post-nonclassical paradigm, according to Stepin, requires reflection on the value-goal orientations of the subject — precisely what is formalized in ODTOE through the parameters (B, A, H) of the observer’s state vector.

From the perspective of the philosophy of science, the transition from π as a geometric constant to π as an invariant of observation represents a paradigm shift in the sense of Kuhn [24]: what changes is not a particular model but the basic ontological category that determines the status of fundamental mathematical objects in physical theory.

6.2. Limitations

The boundaries of the proposed analysis should be explicitly stated.

First, the arguments of Sections 3.1 and 3.2 rely on assumptions (D-Top) and (D-Fr), the rigorous justification of which requires specification of the analytical properties of the operators $\hat{O}$ and ι. This task is designated as open in the main ODTOE article [3, Section II].

Second, the argument of Section 3.5 (Euler's identity) is conceptual rather than probative in character. It points to structural consistency but is not an independent derivation.

Third, the interpretation of the Planck constant (Section V) remains heuristic. For the transition to testable predictions, specification of the functional F (equation 4.6 [3]) and experimental determination of the theory's parameters are necessary.

Fourth, the connection between the transcendence of π and structural incompleteness (Section 4.1) is formulated as a consistency argument, not as a theorem. A rigorous derivation would require defining the class of admissible closure constants and proving that algebraic numbers are excluded from this class.

Fifth, the arguments of Section V-bis on the complementarity of π and φ rely on structural parallels (the common mechanism of Banach's theorem, the duality of continuous and discrete spectra); however, a quantitative relationship between the two invariants within a single formula has not been established. Formalizing such a connection would require specification of the full nonlinear dynamics of the system $R \leftrightarrow B$, including regimes in which continuous and discrete dynamics interact.

VII. CONCLUSION

It has been shown that the number π naturally appears in the ODTOE formalism through five independent arguments: topological (the closure of the self-observation loop generates the homotopy invariant 2π), spectral (the imaginary part of the eigenvalues of the operator Φ contains 2π as the condition for a full phase cycle), measure-theoretic (the normalization of the Gaussian measure on the infinite-dimensional space of potential states includes the factor $\sqrt{2\pi}$), dynamical (the oscillation period of the coupled system $R \leftrightarrow B$ contains 2π), and algebraic (Euler's identity connects all key elements of the formalism).

The transcendence of π is interpreted as a formal expression of the structural incompleteness of ODTOE: the spiral (rather than circular) dynamics of observation ensures the inexhaustibility of reality. The ternary architecture of the minimal observation act (observer, observed, operator) is connected to Archimedes' lower bound $\pi > 3$. It has been established that the strange loop represents a topologically distinguished mechanism of self-generation of reality.

A substantive interpretation of the factor 2π in the Planck constant as the quantitative expression of one full cycle of self-observation has been proposed. It has been shown that the golden ratio $\varphi = (1 + \sqrt{5})/2$ serves as a complementary structural invariant generated by the discrete iterative dynamics of self-reference through the same mechanism of Banach's theorem [7], which establishes the existence of the fixed point Ψ^*. The two invariants — π and φ — govern different aspects of the dynamics: π governs continuous phase rotations, φ governs discrete iterations and orbital stability.

Further work envisages: (a) rigorous specification of the analytical properties of the operators $\hat{O}$ and ι, necessary for the formalization of assumptions D-Top and D-Fr; (b) numerical modeling of the spiral dynamics near the fixed point; (c) investigation of the connection between the class of transcendental numbers and the class of admissible closure constants of self-referential systems; (d) formalization of the interaction of π- and φ-invariants in the full nonlinear dynamics.

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